



MAX-PLANCK-INSTITUT
FÜR KERNPHYSIK

Advanced Lab Course

Data Analysis with KATRIN

Neutrino Mass and Sterile Neutrino Searches

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1 Introduction

Neutrinos play a key role in answering some of the big open questions in physics. The particle was postulated in 1930 by Wolfgang Pauli to describe the continuous electron energy spectrum observed in β -decay experiments. Over 20 years later, in 1956, it was experimentally confirmed by the Cowan-Reines neutrino experiment [1].

Since its discovery the understanding of the neutrino has increased tremendously and it is now well embedded in the Standard Model of particle physics. Neutrinos participate in the weak interaction, one of the three fundamental forces described by the Standard Model and the one responsible for β -decays. In correspondence with the charged leptons, the electron e , the muon μ and the tau τ , neutrinos come in three different flavours ν_e , ν_μ and ν_τ .

Various experiments have proven that it is possible for neutrinos to change their flavour by neutrino oscillations [2–4], which arise from the fact that the neutrino mass eigenstates $|\nu_1\rangle$, $|\nu_2\rangle$ and $|\nu_3\rangle$ are not equal to the neutrino flavour eigenstates $|\nu_e\rangle$, $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$. Each flavour state is composed of the three different mass eigenstates with a unique mixing described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [5, 6]:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (1.1)$$

The probability to oscillate from one state α into another state β^*

$$P_{\alpha \rightarrow \beta} \propto \sin^2 \left(\frac{\Delta m_{12}^2 L}{4E} \right) \quad (1.2)$$

shows that neutrino oscillation experiments are only sensitive to the difference of the squared masses of two different mass states $\Delta m_{12}^2 = m_2^2 - m_1^2$. The existence of neutrino oscillations hence proves that neutrinos have mass, however the absolute mass scale of the neutrinos cannot be deduced from oscillation experiments.

Currently, there are three main methods to determine the absolute neutrino mass scale: cosmological investigations of large scale structure (LSS) formation and evolution [7], the search for neutrinoless double β -decay ($0\nu\beta\beta$) [8] and the analysis of the shape of the energy spectrum of electrons emitted by the β -decay near the endpoint.

While the LSS analysis depends heavily on the cosmological model being used and $0\nu\beta\beta$ requires the neutrino to be its own anti-particle, a so-called Majorana particle, the kinematics of the β -decay do not depend on the neutrino mass model and are based on well-known physics. The **K**ARlsruhe **T**RItium **N**eutrino (KATRIN) experiment is designed to explore the absolute neutrino mass scale using this method with unprecedented sensitivity. The current best direct laboratory limit stems from KATRIN with $m_\nu < 0.45$ eV [9].

Another interesting characteristic of the neutrino is its chirality - which for this purpose can be described as the projection of the spin onto the direction of momentum[†]. The Goldhaber experiment revealed that neutrinos in nature exist exclusively in left-chiral states, while anti-neutrinos are found only in right-chiral states [10]. This stands in contrast to other elementary fermions, which possess both left- and right-handed components, suggesting the potential existence of additional right-handed neutrinos and left-handed anti-neutrinos, which is well motivated by theory: they could explain e.g. neutrino mass generation and the matter-antimatter asymmetry. If their masses lie in the keV range, they also emerge as compelling dark matter candidates [11]. To match experimental observations —

*For a clearer introduction to neutrino oscillations, the simplified case of only two different neutrino flavours $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle$ with $|\nu_1\rangle$ and $|\nu_2\rangle$ is shown here.

[†]This statement is exact only for massless particles, for which chirality and helicity coincide; for massive particles chirality is a Lorentz-invariant quantum number distinct from helicity.

such as the detection of solely left-chiral neutrino states in nature in the Goldhaber experiment and the precise constraint on the active light neutrino species $N_\nu \approx 3$ by studying the Z_0 decay width [12] - these states must be sterile. This means that they do not participate in the weak interaction and interact only via gravity.

To incorporate sterile neutrinos into the Standard Model with minimal extension, the so-called 3+1 neutrino model introduces a fourth, sterile neutrino ν_s that mixes with the three active neutrinos [13]. The extended PMNS matrix takes the form

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}, \quad (1.3)$$

where $|\nu_4\rangle$ is the sterile mass eigenstate. The mixing amplitude between $|\nu_4\rangle$ and electron neutrinos is parametrized by $|U_{e4}|^2$ (for muon and tau neutrinos the amplitude is $|U_{\mu4}|^2$ and $|U_{\tau4}|^2$, respectively).

Since all elements of the extended PMNS matrix are constrained by the unitarity condition, high-precision measurements of the active neutrino sub-matrix limit the active-sterile mixing parameters. Sterile neutrinos can also be probed in β -decay experiments [14, 15], where the sterile mass will distort the energy spectrum.

The KATRIN experiment was designed to measure the absolute neutrino mass, but its data also enables the search for a fourth, sterile neutrino through its characteristic shape distortion to the β -spectrum. With this approach, KATRIN has been able to exclude a large region of the sterile-neutrino parameter space, complementary to constraints from oscillation experiments [16].

In this data analysis course we will first understand the basic model of the KATRIN experiment. Equipped with this, we can then investigate how the absolute value of the neutrino mass impacts the β -decay spectrum and thus how KATRIN attempts to infer its value. We will come to the concepts of interval and uncertainty estimation as well as the idea of the sensitivity of an experiment. Finally, we will extend our initial model to include a hypothetical sterile neutrino and analyze the data for possible signatures of its presence.

To this end, the lab course is divided in three main parts:

1. The first part of the lab course focuses on the KATRIN experiment. Here your task will be to program a simplified model of the tritium β -decay spectrum and to graphically illustrate the impact of the neutrino mass on the shape of the spectrum. You will then implement KATRIN-specific experimental effects (such as energy resolution) in your model.
2. In the second part you will focus on data-analysis. You will learn how to infer the physics parameter of interest, in our case the neutrino mass, from a fit to the data. You will also learn how to correctly evaluate the uncertainty of the fit result and set an upper limit in case no neutrino mass is found.
3. The third part is dedicated to the search for a sterile neutrino signature. You will perform a grid scan in the two-dimensional sterile neutrino parameter space to test for the presence of an additional sterile state. Based on this scan, you will derive exclusion limits. In addition, you will simulate a sterile neutrino signal and investigate how its presence manifests itself in the grid scan.

2 The KATRIN Experiment

The **K**Arlsruhe **T**Ritium **N**eutrino (KATRIN) experiment is a tritium β -decay experiment following the measurement principle of its predecessors in Mainz [17] and Troitsk [18]. It is designed to measure the effective electron anti-neutrino mass

$$m_\nu = \left(\sum_{i=1}^3 |U_{ei}|^2 \cdot m_i^2 \right)^{\frac{1}{2}} \quad (2.1)$$

with a sensitivity of $< 0.3 \text{ eV}^\dagger$ at 90 % confidence level (C.L.).

This chapter gives an overview of the measurement principle of the KATRIN experiment with a focus on deriving a model used for both analyses, the neutrino mass determination and the search for a sterile neutrino. Much more details on the experimental setup can be found in the KATRIN design report [19], the models builds upon [20].

2.1 Neutrino Mass Determination from Beta-Decay

In a β^- -decay process a neutron in a nucleus X is transformed into a proton leaving over the daughter nucleus Y and emitting an electron e^- , an electron anti-neutrino $\bar{\nu}_e$ and the surplus energy Q :

$$X \rightarrow Y + e^- + \bar{\nu}_e + Q. \quad (2.2)$$

The released energy Q is shared between the decay products. The daughter nucleus Y is left with the recoil energy E_{rec} while the remaining energy, called the endpoint E_0 , is split between the electron and the anti-neutrino with corresponding energies E and E_ν respectively:

$$E_0 = Q - E_{\text{rec}} = E + E_\nu \quad (2.3)$$

As the energy of the neutrino E_ν is given by

$$E_\nu = \sqrt{m_\nu^2 + p_\nu^2}, \quad (2.4)$$

a neutrino with non-zero rest mass always takes some of the released energy and hereby reduces the maximal energy E the electron can receive. Equation (2.5) gives the differential rate for an allowed β -decay in dependence of the electron energy E [21]:

$$\frac{d\Gamma}{dE}(E; m_\nu^2) = C \cdot F(Z', E) \cdot p \cdot (E + m_e) \cdot (E_0 - E) \cdot \sqrt{(E_0 - E)^2 - m_\nu^2} \cdot \Theta(E_0 - E - m_\nu). \quad (2.5)$$

Here the constant

$$C = \frac{G_F^2 \cos^2 \theta_C}{2\pi^3} \cdot |M_{\text{nuc}}|^2 \quad (2.6)$$

incorporates the Fermi constant G_F , the Cabbibo angle θ_C and the nuclear matrix element M_{nuc} . $F(Z', E)$ is the Fermi function that accounts for electromagnetic interactions of the outgoing electron with the daughter nucleus and the Heaviside function Θ assures energy conservation. This dependence is shown in fig. 1a while the impact of a non-zero neutrino mass on the β -spectrum shape in the endpoint region is displayed in fig. 1b.

[†]We use natural units ($\hbar = c = 1$) for better readability

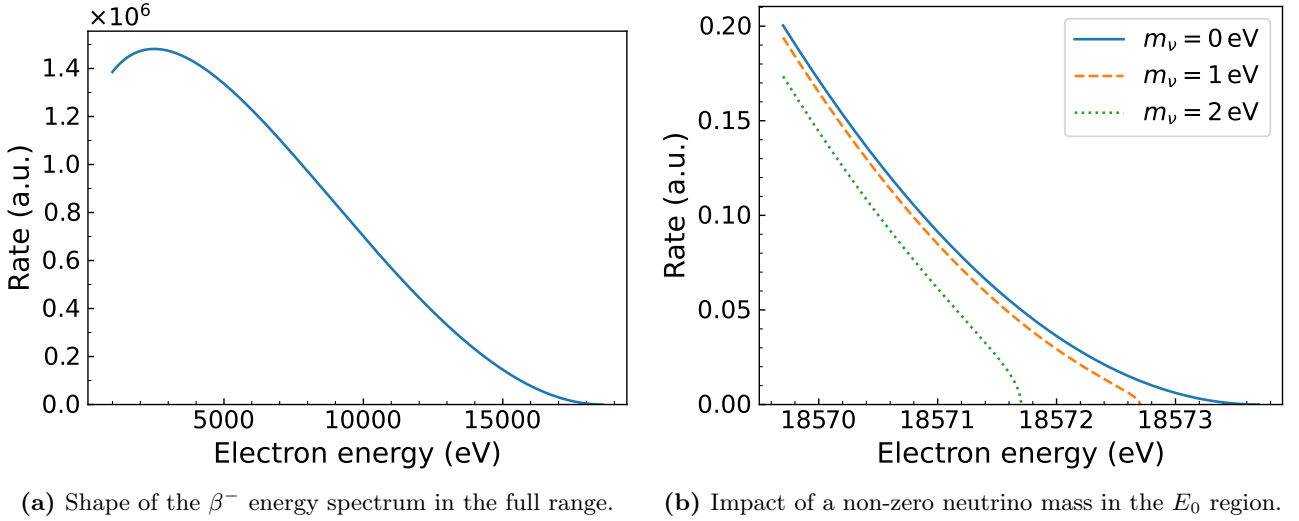


Figure 1: Differential tritium β -spectrum in dependence of the electron energy.

As one can deduce from fig. 1b, measuring the differential spectrum in the endpoint region with sub-eV precision is an option to determine the absolute neutrino mass scale. From the magnitude of the impact it is clear that one requires an experiment with excellent energy resolution in the eV range and the very low rates in the endpoint region show that a highly luminous source is needed. Both requirements are fulfilled in the KATRIN experiment and are explained in section 2.3.

2.2 Sterile Neutrino Search with Beta-Decay

As already mentioned, KATRIN's data also enables the search for a fourth, sterile neutrino through its characteristic shape distortion to the β -spectrum. In the 3+1 neutrino model, the sterile neutrino is parameterized by its mass, m_4 , and mixing to the electron flavor, $\sin^2 \theta^s$. The tritium β -spectrum from eq. (2.5) modifies to

$$\frac{d\Gamma}{dE}(E; m_\nu^2, m_4^2, \sin^2 \theta) = (1 - \sin^2 \theta) \cdot \frac{d\Gamma}{dE}(E, m_\nu^2) + \sin^2 \theta \cdot \frac{d\Gamma}{dE}(E, m_4^2). \quad (2.7)$$

The sterile neutrino signature is a global distortion of the β -spectrum for energies $E < E_0 - m_4$. A sharp kink marks the onset of the sterile neutrino contribution. The impact of a sterile neutrino on the β -decay spectrum is illustrated in fig. 2, assuming a mass of $m_4^2 = 10 \text{ eV}^2$ ($m_4 = 3.2 \text{ eV}$) and a large mixing of $\sin^2 \theta = 0.2$.

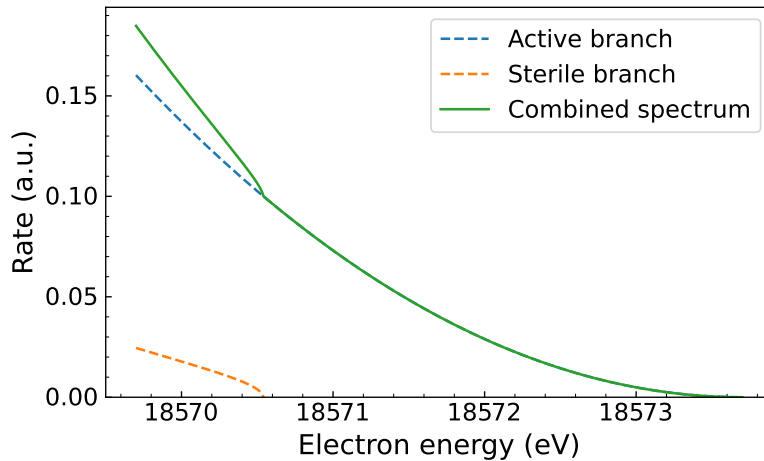


Figure 2: Kink-like impact of a sterile neutrino on the β -decay spectrum.

[§]In the following the mixing amplitude $|U_{e4}|^2$ from eq. (1.3) is parametrized as $\sin^2 \theta$.

2.3 Experimental Setup

An overview of the experimental setup is given in fig. 3. Each element is briefly described in the following paragraphs.

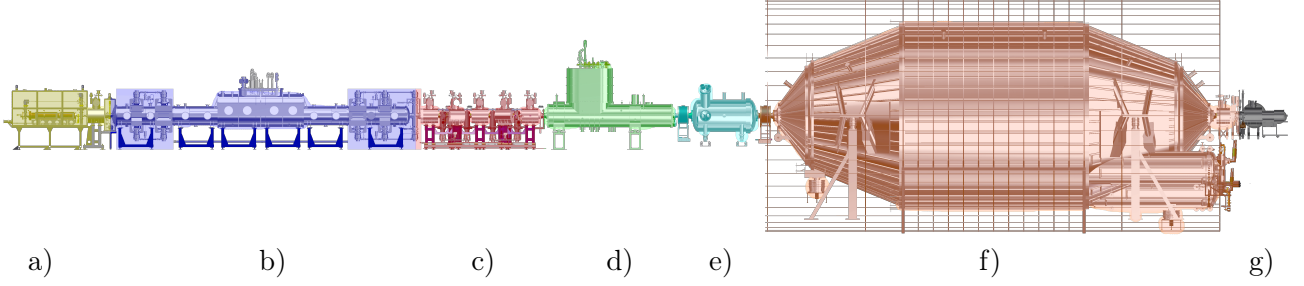


Figure 3: Overview of the KATRIN experiment with the main components:

- a) Rear section
- b) Windowless gaseous tritium source (WGTS)
- c) Differential pumping section (DPS)
- d) Cryogenic pumping section (CPS)
- e) Pre-spectrometer (PS)
- f) Main-spectrometer (MS)
- g) Focal plane detector (FPD)

2.3.1 Rear Section

The main purpose of the rear section is to supply a defined electric potential of the tritium gas in the source by providing a conducting surface, the rear wall. In addition it houses monitoring and calibration tools [22].

2.3.2 Windowless Gaseous Tritium Source

The windowless gaseous tritium source (WGTS) [23] is an ultra stable and highly luminous gaseous tritium source consisting of a stainless steel tube of 10 m length and 90 mm diameter. Tritium gas of high purity is continuously injected into the source and streams freely to both ends, where it is pumped away and then fed into a sophisticated loop system which reprocesses and re-injects the tritium.

Tritium as a β -emitter has various advantageous properties for a β -decay experiment [24]:

- Its endpoint value of $E_0 \approx 18.6 \text{ keV}$ [21] is the second lowest of all isotopes undergoing β -decay. A low endpoint value is advantageous as it leads to relatively more counts in the endpoint region in which the neutrino mass manifests itself.
- The tritium decay is a super-allowed transition which leads to a rather short half-life of $T_{1/2} = 12.3 \text{ years}$ with correspondingly high rates at low source densities.
- In its molecular form T_2 , tritium can be used in its gaseous state at low temperatures. This state is preferred as systematic uncertainties are decreased and higher rates are achievable.

Since KATRIN uses tritium in its molecular form, the tritium β -decay is described by



2.3.3 Pumping Sections

Tritium from the source may not reach the spectrometers as decay processes in the spectrometers would be a major source of background. The task of the pumping sections is to reduce the tritium flow rate by 14 orders of magnitude in combination with the source which already achieves a reduction by a factor of 100.

2.3.4 Pre- and Main-Spectrometer

There are two spectrometers installed in the KATRIN beamline, the pre- and the main-spectrometer. Both are built following the so-called MAC-E filter principle (magnetic adiabatic collimation and an electrostatic high-pass filter) depicted in fig. 4.

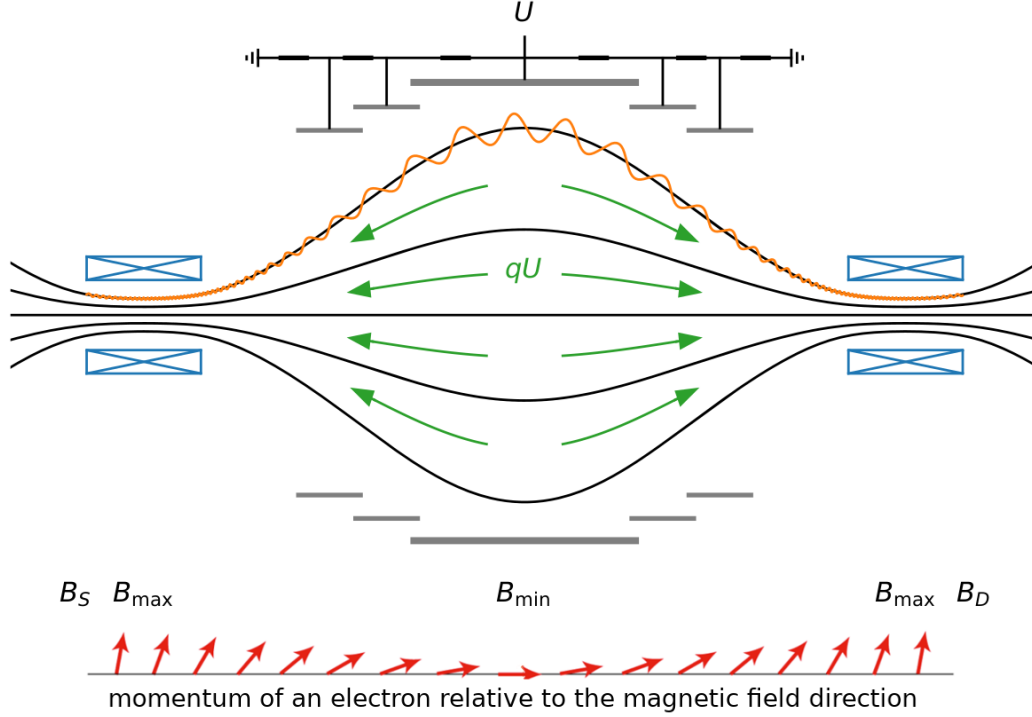


Figure 4: Principle of the MAC-E filter, figure adapted from [25] and [26].

Electrons emitted in the source are transported adiabatically by a magnetic field towards the centre of the main spectrometer. There they have to overcome the retarding potential qU ($q = -1e$ with the electric charge of the electron e), shown in green. Only electrons with greater kinetic energy than this retarding energy can reach the detector. Hence, the spectrometer acts as a high-pass filter for the electron energy. The retarding voltage U , usually around ≈ 18 kV, applied to the spectrometer can be varied. This results in a measurement of the integral spectrum of the tritium β -decay.

The retarding potential is only sensitive to the longitudinal energy component of the electrons $E_{\parallel}$. Thus, it is crucial that in the analyzing plane, the total kinetic energy of the electrons is collimated into the longitudinal direction:

$$E_{\text{kin}} = E_{\parallel} + E_{\perp} \approx E_{\parallel}. \quad (2.9)$$

The residual transversal component $E_{\perp}$ defines the energy resolution ΔE of the spectrometer.

To assure eq. (2.9), the momenta of the electrons are collimated magnetically and adiabatically via the inverse magnetic mirror effect [27–29]. The electrons perform a cyclotron motion while travelling along the magnetic field lines (shown in orange). Including relativistic effects, the invariant

$$\frac{p_{\perp}^2}{B} = \frac{E_{\perp} \cdot (\gamma + 1) \cdot m_e}{B} = \text{const.} \quad (2.10)$$

is conserved (with the transversal momentum of the electron $p_{\perp}$, the changing magnetic field B and the relativistic gamma factor $\gamma = \frac{E+m_e}{m_e}$).

The β -electrons are emitted in a region with a high magnetic field ($B_S \approx 2.5$ T). While they are guided to the analyzing plane, the field strength drops by several orders of magnitude ($B_{\min} \approx 6 \times 10^{-4}$ T) and

the transversal energy component is transformed into the longitudinal one by the magnetic gradient force as stated in eq. (2.10). Due to the conservation of the magnetic flux per area the spectrometer (≈ 10 m) must be much larger than the source (≈ 10 cm).

If the source was positioned at the maximum magnetic field, all electrons emitted in direction of the spectrometer would be guided towards the electrostatic barrier, i.e. the acceptance angle for the electrons would be 90° . In practice this is unfavourable as electrons with large starting angles travel long distances in the source which increases their probability to scatter. Therefore the source is placed at a lower magnetic field $B_S < B_{\max}$ leading to a reduced acceptance angle ($\approx 51^\circ$) by the magnetic mirror effect.

The energy resolution of the main spectrometer is determined by the magnetic field setting:

$$\frac{\Delta E}{E} = \frac{B_{\min}}{B_{\max}}. \quad (2.11)$$

For the recent KATRIN tritium campaign the maximum magnetic field was $B_{\max} = 4.2$ T leading to an energy resolution of $\Delta E \approx 2.7$ eV at 18.6 keV.

The smaller pre-spectrometer was operated at a retarding energy of around 300 eV less than the main spectrometer. It thus acted as a pre-filter rejecting the majority of uninteresting electrons that would not make it past the main spectrometer for background reduction. But because the combination of two high-voltage MAC-E filters in series leads to the formation of a so-called Penning trap, which causes an additional background, it is no longer operated at high voltages.

2.3.5 Focal Plane Detector

The electrons that pass the main spectrometer are re-accelerated due to the MAC-E filter and after further guidance detected by the focal plane detector (FPD) [30], which is a multi-pixel silicon semiconductor detector. The 148 pixels of equal area are arranged in a dart-board shape as shown in fig. 5. Each pixel counts the number of electrons detected.

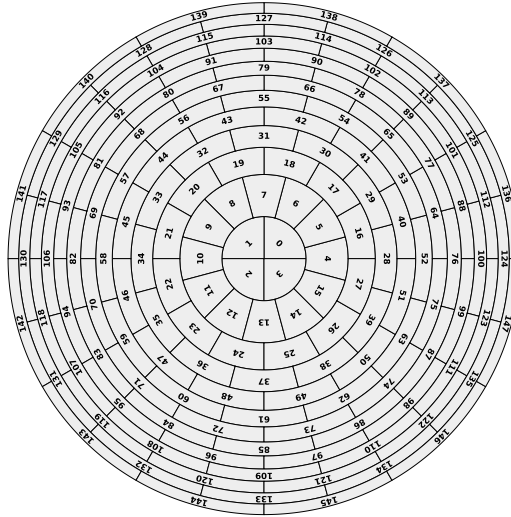


Figure 5: Pixel segmentation of the focal plane detector.

2.4 Modelling of the Integrated Beta-Decay Spectrum

This section focusses on deriving a model of the integrated β -decay spectrum describing the expected rate of the KATRIN experiment. First, two effects modifying the differential rate $\frac{d\Gamma}{dE}$ are briefly explained. Afterwards the response of the experimental setup is discussed and finally all components are combined to a complete model of the rate expectation.

2.4.1 Final States Distribution

As described in section 2.3.2, KATRIN uses tritium in its molecular form as β -decay source. Therefore the daughter molecule can end up in a rotational-vibrational or electronic excited state with energy V_f and corresponding probability P_f . This modifies eq. (2.5) as the effective maximum energy available for electron and neutrino shifts away from the endpoint: $E_0 \rightarrow E_0 - V_f$. In this course, this effect will be neglected.

2.4.2 Doppler Effect

Additionally, since the tritium molecules in the source are in thermal motion, the differential energy spectrum is broadened by the Doppler effect. This effect will also be neglected in this course.

2.4.3 Response Function

The response function $R(qU, E)$ describes the probability of an electron with energy E to pass the MAC-E filter at a given retarding potential qU . In an ideal apparatus with perfect energy resolution and without energy loss, $R(qU, E)$ would be a simple step function:

$$R(qU, E) = \begin{cases} 0 & E < qU \\ 1 & E \geq qU \end{cases} \quad (2.12)$$

In the KATRIN experiment two effects dominate the response function: the transmission function of the MAC-E filter and energy loss due to scattering effects in the source. The latter effect will only be discussed briefly here.

Not all the perpendicular momentum of an electron is converted to momentum parallel to the electric field in the spectrometer. Therefore also electrons with $E \geq qU$ may not pass the electrostatic barrier even if they would have enough energy. The resulting transmission probability can be described by

$$T(qU, E) = \begin{cases} 0 & E < qU \\ \frac{1 - \sqrt{1 - f \cdot \frac{B_S}{B_{\min}} \cdot \frac{E - qU}{E}}}{1 - \sqrt{1 - \frac{B_S}{B_{\max}}}} & qU \leq E \leq qU \frac{f \cdot B_{\max}}{f \cdot B_{\max} - B_{\min}} \\ 1 & E > qU \frac{f \cdot B_{\max}}{f \cdot B_{\max} - B_{\min}} \end{cases} \quad (2.13)$$

with the relativistic factor

$$f = \frac{\frac{E - qU}{m_e} + 2}{\frac{E}{m_e} + 2}. \quad (2.14)$$

The shape of the transmission function is shown in fig. 6a for typical parameter values during the recent neutrino mass measurements.

The major energy loss component is inelastic scattering of β -electrons with tritium molecules in the source. The probability of an electron to scatter i -times is P_i and the energy ϵ lost by i -fold scattering is described by the energy loss function $f_i(\epsilon)$. While this effect is neglected in this course, it is mentioned here for completeness.

The total response function is then composed as a convolution of the transmission function with the energy loss function.

$$R(qU, E) = \int_0^{E - qU} T(qU, E - \epsilon) \sum_{i=0}^{\infty} P_i \cdot f_i(\epsilon) \, d\epsilon \quad (2.15)$$

An example of the response function is shown in fig. 6b.

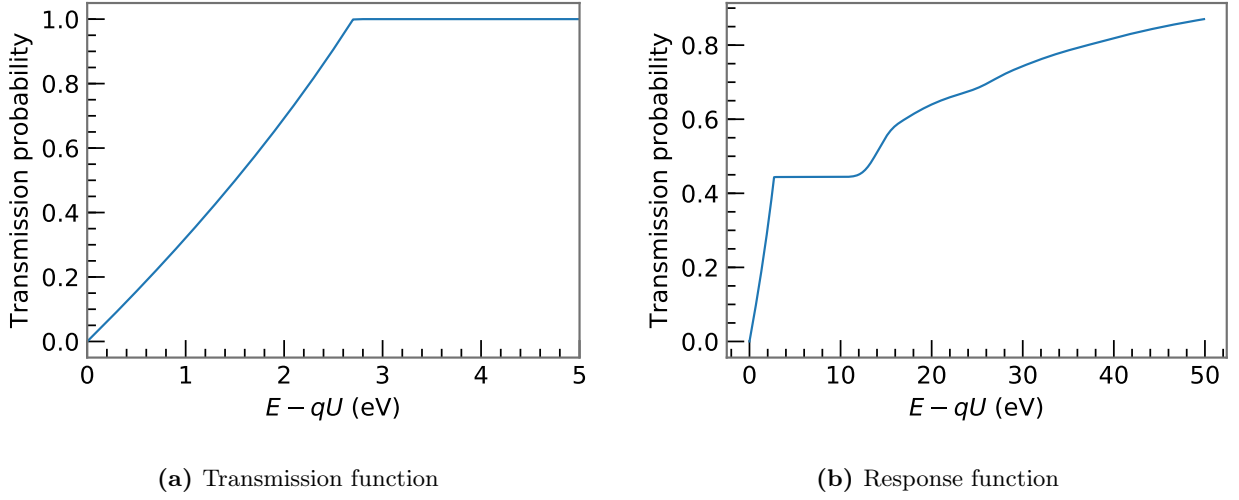


Figure 6: Shape of the transmission and response function.

2.4.4 Model of the Rate Expectation

As mentioned at the beginning of this chapter, KATRIN measures an integrated tritium β -spectrum $I(qU)$. The shape is composed of the differential spectrum $\frac{d\Gamma}{dE}(E)$ (eq. (2.5) for neutrino mass determination and eq. (2.7) for sterile neutrino search) and the response function $R(qU, E)$ derived in eq. (2.15):

$$I(qU) = C \cdot \int_{qU}^{E_0} \frac{d\Gamma}{dE}(E) \cdot R(qU, E) dE \quad (2.16)$$

with a constant prefactor C .

In addition to the signal $I(qU)$, a constant background rate B is expected and the integral spectrum is multiplied by a signal amplitude A_{sig} , which accounts for the number of tritium molecules in the source, the acceptance angle and other normalization effects. The total rate $\Gamma(qU)$ is then

$$\Gamma(qU) = A_{\text{sig}} \cdot I(qU) + B \quad (2.17)$$

Figure 7 shows an integral β -spectrum as example.

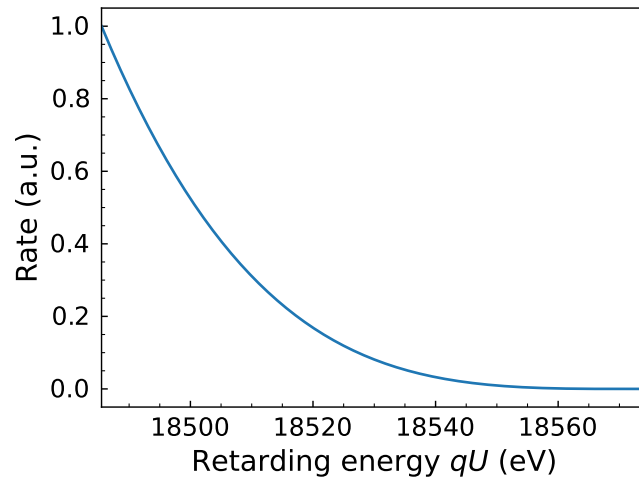


Figure 7: Integral tritium β -spectrum measured with the KATRIN experiment, figure adapted from [31].

3 Basic Principles of Data Analysis

In this chapter we will cover some of the basic principles of data analysis. We will make heavy use of these during the lab course, so make sure to read this carefully. In case there are any questions or something is unclear, we can discuss this before starting the actual tasks.

3.1 Maximum Likelihood Analysis

The likelihood function $\mathcal{L}$ is used in statistical methods of parameter inference. It describes how probable a specific outcome of an experiment x is given a model prediction μ . The model itself can depend on a set of parameters θ . The likelihood function then reads as

$$\mathcal{L} = \mathcal{L}(\mu(\theta); x) = \mathcal{L}(\theta; x). \quad (3.1)$$

To retrieve the model that best describes the data, a set of parameters θ is found such that it maximizes the likelihood function.

We now infer the basic likelihood function describing KATRIN data. As described in section 2, in KATRIN an integrated tritium β -decay spectrum is measured. As the decay process itself is stochastic, the number of expected decays and hence the number of expected electron counts is described by a probability distribution function (PDF). Assuming the decay rate in the source is constant and the individual decays are independent of one another, the underlying PDF of the number of decays is the Poisson distribution (eq. (3.2)). It describes the probability to observe N counts given a model prediction $\mu(\theta)$:

$$P(N \text{ counts}; \mu(\theta)) = e^{-\mu(\theta)} \frac{\mu^N(\theta)}{N!}. \quad (3.2)$$

Thus $\mathcal{L}(\mu(\theta); N) = P(N; \mu(\theta))$ is the likelihood function of the parameters θ for a single point of the integrated tritium β -spectrum. The model prediction μ includes the differential tritium spectrum and modelling of the KATRIN apparatus as described in eq. (2.17).

The entire spectrum consists of many counts N_i measured at given different retarding energies qU_i . The individual points are statistically independent and therefore the joint probability distribution P_{total} for the whole spectrum is the product of the individual PDFs

$$P_{\text{total}}(\vec{N}; \vec{\mu}) = \prod_i e^{-\mu_i} \frac{\mu_i^{N_i}}{N_i!}, \quad (3.3)$$

where i runs over all retarding energies and μ_i is defined as $\mu(qU_i; \theta)$.

In practice it is numerically favorable to minimize $-\ln \mathcal{L}$ instead of maximizing $\mathcal{L}$ as the product of many small numbers quickly leads to numerical difficulties. In addition, taking derivatives of a sum is much easier than taking derivatives of a product.

For large model predictions μ the Poisson distribution asymptotically tends to the normal distribution with mean μ and standard deviation $\sqrt{\mu} \approx \sqrt{N}$:

$$P(N \text{ counts}; \mu(\theta)) = \frac{1}{\sqrt{2\pi N}} e^{-\frac{(N-\mu(\theta))^2}{2N}}. \quad (3.4)$$

In this case minimizing $-\ln \mathcal{L}$ is equivalent to the χ^2 -minimization:

$$\begin{aligned}
-\ln \mathcal{L}(\vec{\mu}; \vec{N}) &= -\ln \left(\prod_i \frac{1}{\sqrt{2\pi N_i}} e^{-\frac{(N_i - \mu_i)^2}{2N_i}} \right) \\
&= -\sum_i \ln \left(\frac{1}{\sqrt{2\pi N_i}} e^{-\frac{(N_i - \mu_i)^2}{2N_i}} \right) \\
&\propto -\sum_i \ln \left(e^{-\frac{(N_i - \mu_i)^2}{2N_i}} \right) \\
&= \sum_i \frac{(N_i - \mu_i)^2}{2N_i} =: \frac{1}{2} \chi^2(\vec{\mu}; \vec{N}).
\end{aligned}$$

In a basic KATRIN model, θ consists of four free parameters: the neutrino mass squared m_ν^2 , the endpoint value E_0 , the normalization of the spectrum A_{sig} and the constant background rate B :

$$\mu_i(qU_i, t; m_\nu^2, E_0, A_{\text{sig}}, B) = \left(A_{\text{sig}} \cdot \int_{qU_i}^{E_0} \frac{d\Gamma(E; m_\nu^2, E_0)}{dE} \cdot R(qU, E) dE + B \right) \cdot t. \quad (3.5)$$

For the sterile neutrino search, there are two additional parameters, m_4^2 and $\sin^2 \theta$:

$$\mu_i(qU_i, t; m_\nu^2, E_0, A_{\text{sig}}, B, m_4^2, \sin^2 \theta) = \left(A_{\text{sig}} \cdot \int_{qU_i}^{E_0} \frac{d\Gamma(E; m_\nu^2, E_0, m_4^2, \sin^2 \theta)}{dE} \cdot R(qU, E) dE + B \right) \cdot t. \quad (3.6)$$

3.2 Interval Estimation

In addition to determining the model parameters that best describe our data, we are also interested in estimating an interval that characterizes the range of plausible parameter values. The simplest such characterization is the 1- σ uncertainty on a parameter, which can be obtained by constructing a so-called profile likelihood.

Let the best-fit parameter set θ_{best} yield a minimum value χ_{best}^2 . To determine the 1- σ uncertainty for m_ν^2 , we proceed as follows:

1. For fixed values of m_ν^2 , perform fits with the remaining parameters $\hat{\theta} = (E_0, A_{\text{sig}}, B)$ left free, and record the resulting value $\chi^2(m_\nu^2)$.
2. Compute the difference $\Delta\chi^2(m_\nu^2) = \chi^2(m_\nu^2) - \chi_{\text{best}}^2 \geq 0$.
3. Identify the values of m_ν^2 for which $\Delta\chi^2(m_\nu^2) = 1 \equiv \Delta\chi_{\text{crit}}^2$.

This will in general give us two values, one smaller and one larger than our best fit value of m_ν^2 , see fig. 8.

In a frequentist analysis the estimated intervals describe so-called confidence intervals. An interval at 90 % confidence level contains the true value in 90 % of the cases. Note, that it says nothing about the true value of the one experiment performed. Instead it says if we were to repeat the experiment $N \rightarrow \infty$ times, then 90 % of our constructed intervals would contain the true value.

The interval $[m_{\nu, \text{lower}}^2, m_{\nu, \text{upper}}^2]$ which we retrieve from our profile likelihood analysis is given at 1- σ confidence level which corresponds to 68.27 %. If we assume our parameter estimates are normally distributed, which thanks to the central limit theorem is often the case, we can apply scaling factors to come to other common confidence intervals. For example, the 90 % confidence interval is retrieved by multiplying σ with 1.645 and the 95 % confidence interval by multiplying with 1.960.

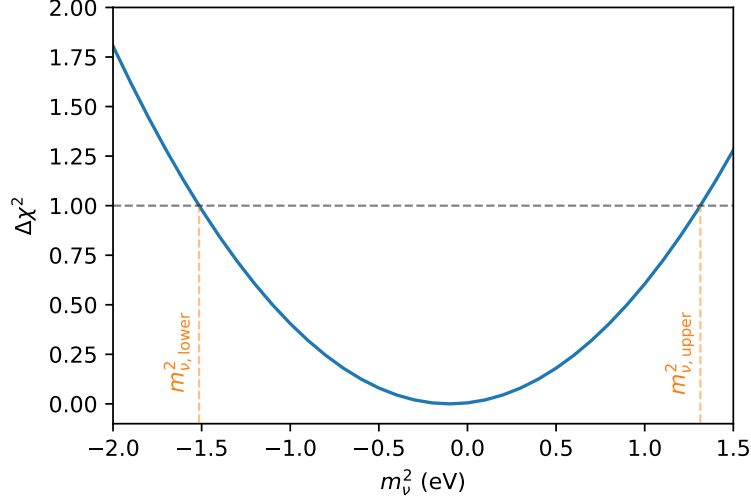


Figure 8: Profile likelihood with respect to m_ν^2 . We retrieve two values fulfilling $\Delta\chi^2 = 1$:
 $m_{\nu,\text{lower}}^2 = m_{\nu,\text{best}}^2 - \sigma_{\text{lower}} < m_{\nu,\text{best}}^2 = 0 \text{ eV}^2 < m_{\nu,\text{upper}}^2 = m_{\nu,\text{best}}^2 + \sigma_{\text{upper}}$.
Often the problems analysed are symmetric and we can simplify to $\sigma = \sigma_{\text{lower}} = \sigma_{\text{upper}}$.

A brief explanation of the last step of the profile likelihood construction is given by Wilks' theorem [32]. Under fairly general conditions the test statistic $\Delta\chi^2(m_\nu^2) = \chi^2(m_\nu^2) - \chi_{\text{best}}^2$ follows a χ^2 distribution. Since only a single parameter (m_ν^2) is varied while all other parameters are profiled over, $\Delta\chi^2(m_\nu^2)$ follows a χ^2 distribution with one degree of freedom, which is shown in fig. 9a.

For one degree of freedom, the value $\Delta\chi_{\text{crit}}^2 = 1$ corresponds to a probability of 68.27 %. This comes from the cumulative distribution function (CDF) F of the χ^2 distribution with one degree of freedom, which gives the probability that $\Delta\chi^2(m_\nu^2)$ is less than or equal to a given value. In other words, the CDF evaluated at 1 satisfies $F_{\chi_1^2}(1) = 0.6827$, so setting $\Delta\chi^2(m_\nu^2) = \Delta\chi_{\text{crit}}^2 = 1$ selects the central region containing 68.27 % of the probability (see fig. 9b).

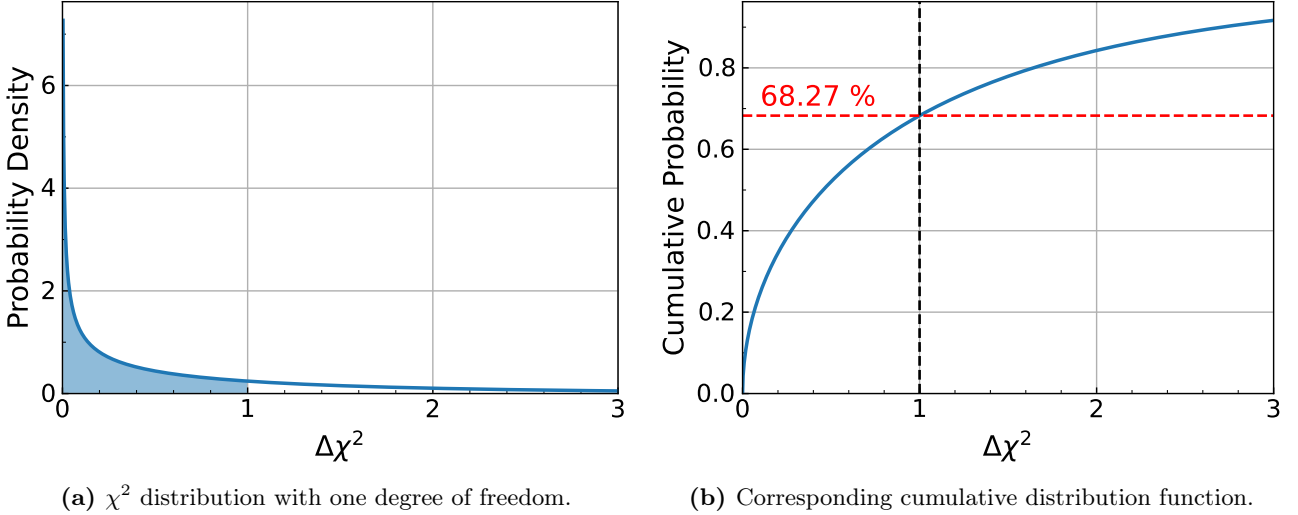


Figure 9: χ^2 distribution with one degree of freedom and its cumulative distribution function, illustrating that $\Delta\chi^2 = 1$ corresponds to a cumulative probability of 68.27 %.

3.3 Sterile Neutrino Search

While the determination of the active neutrino mass involves a single parameter fit, searching for sterile neutrinos requires a two-dimensional exploration. This is because the sterile neutrino signal depends on both the mass squared m_4^2 and the mixing amplitude $\sin^2 \theta$, which together determine the spectral distortion in the integrated β -decay spectrum. Consequently, a two-dimensional parameter space must

be systematically scanned to identify potential signatures or set exclusion limits.

In KATRIN, this is accomplished using a grid search over the sterile neutrino parameter space ($\sin^2 \theta$ on the x -axis and m_4^2 on the y -axis). The procedure consists of the following steps:

1. Define a grid in the sterile neutrino parameter space $[m_4^2, \sin^2 \theta]$. In typical analyses, the grid is chosen to be logarithmically spaced to efficiently cover several orders of magnitude in both parameters. A standard grid consists of 50×50 points.
2. At each grid point, $[m_4^2, \sin^2 \theta]$, perform a fit of the remaining model parameters (E_0, A_{sig}, B)[¶], while keeping the sterile neutrino parameters fixed. This yields the global χ^2 for that specific point.
3. Determine the best-fit parameters $[m_4^2, \sin^2 \theta]_{\text{best}}$ that minimize the global χ^2 over the entire grid: $\chi_{\text{best}}^2 = \chi_{\text{min}}^2$.
4. Construct confidence regions by drawing contours at the isolines $\chi^2 = \chi_{\text{best}}^2 + \Delta\chi_{\text{crit}}^2$. This is similar to the procedure described in section 3.2. Here, one has two degrees of freedom, since two parameters are varied. This leads to $\Delta\chi_{\text{crit}}^2 = 2.30$ for the $1\text{-}\sigma$ confidence level ($\chi_{\text{crit}}^2 = 5.99$ for the 95% C.L.).

This approach provides a systematic way to set limits on sterile neutrino parameters and an example is shown in fig. 10.

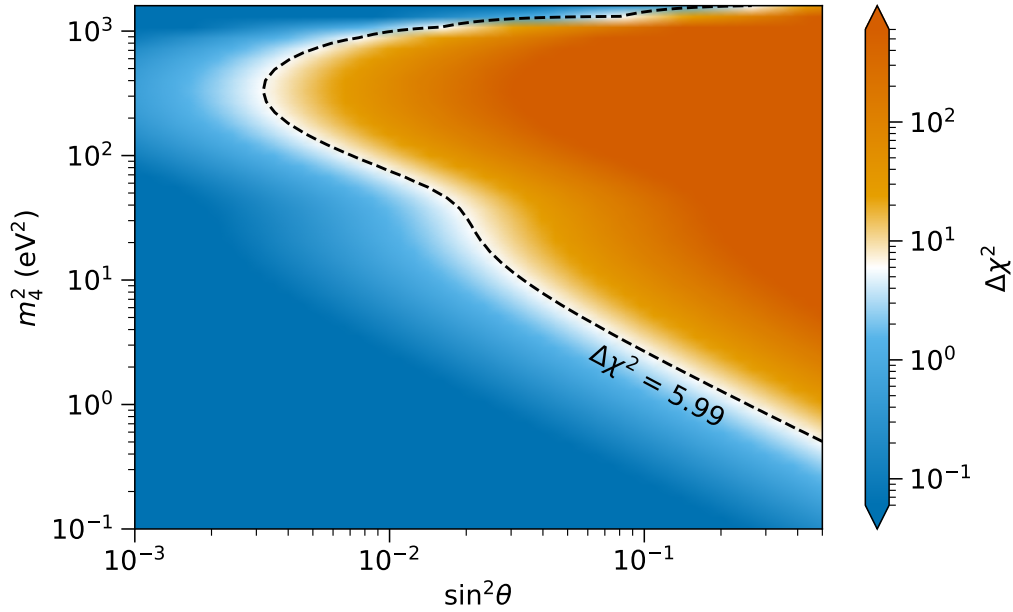


Figure 10: $\Delta\chi^2$ map from a sterile neutrino grid search.

[¶]Setting the active neutrino mass m_ν to zero is a valid approximation assuming that $m_4^2 \gg m_\nu^2$.

4 Tasks

4.1 Understanding the Model

The following tasks all involve writing your own simple code to see the impact of various parameters on the model relevant for the KATRIN experiment. You can pick any programming language of your choice, but I will only be able to help you in python.

4.1.1 Differential Spectrum

Write your own differential tritium spectrum according to eq. (2.5). You can set the constant prefactor to $C = 1.433488 \times 10^{-13}$. You can use the following approximation of the relativistic fermi function:

$$\begin{aligned} E_{\text{tot}} &= E + m_e && \text{total electron energy} \\ p &= \sqrt{E_{\text{tot}}^2 - m_e^2} && \text{electron momentum} \\ \beta &= \frac{p}{E_{\text{tot}}} && \text{relativistic beta factor} \\ \eta &= \frac{\alpha \cdot Z'}{\beta} && \text{number of protons in T}_2 \text{ } Z', \text{ fine structure constant } \alpha \end{aligned}$$

$$F(Z', E) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \cdot (1.002037 - 0.001427\beta).$$

Hint: Use natural units ($c = 1$).

Plot the differential spectrum for $E_0 = 18573.7 \text{ eV}$ and different neutrino masses $m_\nu = 0 \text{ eV}$, 1 eV and 2 eV in the energy range of

- 1000 eV to 18573.7 eV
- 18535 eV to 18573.7 eV
- 18569.7 eV to 18573.7 eV.

What happens if you insert a negative value for m_ν^2 , for example $m_\nu^2 = -1 \text{ eV}^2$? How could this be interpreted statistically? We can spend some time discussing this, there is no need to have an answer immediately.

4.1.2 Sterile Neutrino Extension

Extend the differential spectrum from above by an additional sterile neutrino as in eq. (2.7).

Plot the extended differential spectrum for

- $E_0 = 18573.7 \text{ eV}$
- $m_\nu = 0 \text{ eV}$
- $m_4^2 = 10 \text{ eV}^2$ and
- $|U_{e4}|^2 = 0.2$
- in the energy range of 18569.7 eV to 18573.7 eV.

4.1.3 Transmission function

Write your own analytical transmission function for the MAC-E filter according to eq. (2.13). For the magnetic fields, use the values $B_S = 2.52 \text{ T}$, $B_{\text{max}} = 4.23 \text{ T}$, $B_{\text{min}} = 6.3 \times 10^{-4} \text{ T}$. Plot the transmission

function for reasonable values of qU and E with respect to the analysis window[‡] of KATRIN. What happens if you increase/decrease $B_{\max}$? Or $B_{\min}$? Relate your observations to the energy resolution of the experiment.

4.1.4 Dummy model

Create your own integral spectrum by integrating the differential spectrum over the transmission function for a given retarding energy:

$$I(qU) = \int_{qU}^{E_0} \frac{d\Gamma}{dE}(E; m_\nu^2, E_0) \cdot T(qU, E) dE. \quad (4.1)$$

To complete the model, add a constant background term B and multiply your integral spectrum by a signal amplitude A_{sig} :

$$\Gamma(qU) = A_{\text{sig}} \cdot I(qU) + B. \quad (4.2)$$

For your dummy model you can use $A_{\text{sig}} = 1 \text{ a.u.}$

Repeat the impact of the neutrino mass plot from section 4.1.1 with a background rate of $B = 0.2 \text{ cps}$ ($\text{cps} = \text{counts per second}$). For all plots in this task use the energy range also used later for the analysis, i.e. from $18\,535 \text{ eV}$ to $18\,581 \text{ eV}$. What differences do you observe?

To better understand the impact of the fit parameters on the model, plot:

$$\frac{\Gamma_{\text{ref}}(qU) - \Gamma_{\text{changed}}(qU)}{\Gamma_{\text{ref}}(qU)}. \quad (4.3)$$

As a reference model Γ_{ref} , use the parameter values $m_\nu^2 = 0 \text{ eV}^2$, $E_0 = 18\,573.7 \text{ eV}$, $A_{\text{sig}} = 1 \text{ a.u.}$ and $B = 200 \text{ mcps}$ ($\text{mcps} = \text{milli counts per second}$).

Then repeat the plot for the following parameter changes in Γ_{changed} :

- $m_\nu^2 = 0.5 \text{ eV}^2$
- $m_\nu^2 = 1 \text{ eV}^2$
- $E_0 = 18\,573.6 \text{ eV}$
- $E_0 = 18\,573.8 \text{ eV}$
- $A_{\text{sig}} = 0.9 \text{ a.u.}$
- $A_{\text{sig}} = 1.1 \text{ a.u.}$
- $B = 190 \text{ mcps}$
- $B = 210 \text{ mcps}$

Which energy regions are affected by the parameters?

Finally, for the reference model, figure out at which point in retarding energy the signal to background ratio is 1.

[‡]roughly 40 eV below the endpoint E_0

4.2 Basics of Data Analysis

For simplification and to avoid the computation of the integral for every model evaluation, we will use the differential spectrum with a signal amplitude and a background as model for this section:

$$M(E) = A_{\text{sig}} \cdot \frac{d\Gamma}{dE}(E; m_\nu^2, E_0) + B. \quad (4.4)$$

You can use $m_\nu^2 = 0 \text{ eV}^2$, $E_0 = 18\,573.7 \text{ eV}$, $A_{\text{sig}} = 1 \text{ a.u.}$ and $B = 200 \text{ mcps}$ as standard input.

4.2.1 Monte Carlo Data

We now assume some specific points in retarding energy qU where we plan to measure our model. In KATRIN, this is called the measuring time distribution (MTD), which describes the points in retarding energy at which we measure and how much relative measuring time is spent there. For simplicity, let's assume 24 points linearly spaced between $18\,535 \text{ eV}$ to $18\,581 \text{ eV}$, each with the same time fraction of $\frac{1}{24}$:

$$q\vec{U} = (18535, 18537, 18539, \dots, 18579, 18581) \quad (4.5)$$

$$\vec{t} = (\frac{1}{24}, \frac{1}{24}, \frac{1}{24}, \dots, \frac{1}{24}, \frac{1}{24}). \quad (4.6)$$

As a total measurement time, let's assume $t_{\text{tot}} = 750 \text{ d.}$ To get the expectation of your model, evaluate it at each point in $q\vec{U}$ to retrieve the count rate per second γ and multiply it with the corresponding measuring time $t_{\text{point}} = \frac{t_{\text{tot}}}{24}$ to get the expected number of counts λ . This spectrum, also referred to as the “Asimov spectrum”, represents the expectation of the experiment to our best knowledge. Given our parameter values are exactly correct, we will measure one statistical representation of this spectrum. Instead of measuring the exact expectation λ , we will measure random counts N according to the Poisson distribution from eq. (3.2).

Calculate and plot the Asimov spectrum and retrieve statistically fluctuated spectra, i.e. plug the expected counts λ into a random Poisson distribution and generate a few, $O(10^3)$, of these randomized spectra. Have a closer look at one energy point as an example. Plot a histogram of all fluctuated rates and compare it with the Asimov rate.

4.2.2 Parameter Inference

An important task in data analysis is to retrieve the set of parameter values that best describe the data. As we have plenty of counts in our example, we will approximate the Poisson distribution by a normal distribution and perform a χ^2 minimization as described in section 3.1. You can start with the Asimov spectrum, implement:

$$\chi^2(m_\nu^2, E_0, A_{\text{sig}}, B; \vec{E}, \vec{\Gamma}, \vec{\sigma}). \quad (4.7)$$

For the uncertainty $\vec{\sigma}$, use the relation from section 3.1. Now use your favourite numerical minimizer to minimize the χ^2 function with respect to the fit parameters $m_\nu^2, E_0, A_{\text{sig}}$ and B . Compare the retrieved parameter estimates with the input values you chose. Repeat this for the fluctuated spectra and plot the parameter distributions (histogram) for each of the 4 parameters. What is the mean value of each parameter? What is the standard deviation? Can you give an intuitive explanation of what the mean/standard deviation represents?

4.2.3 Uncertainty and Sensitivity

In addition to the best-fit value, any good physicist is also interested in the uncertainty on this value as described in section 3.2. For this, perform a profile likelihood analysis on the Asimov dataset and retrieve the $1\text{-}\sigma$ confidence interval for m_ν^2 . Convert this into the 90 % confidence interval by applying the appropriate scaling factor. Finally, retrieve the 90 % upper limit of m_ν by calculating $\sqrt{m_{\nu,\text{upper}}^2}$. As we performed this analysis on the Asimov dataset, the one representative to our best knowledge, the intervals calculated correspond to the sensitivity of our experiment.

4.3 Sterile Neutrino Search

We now aim to search for sterile neutrino signatures in the KATRIN data. For this, extend the simplified model from eq. (4.2) by incorporating an additional sterile neutrino as introduced in eq. (2.7). Perform a grid scan for the Asimov dataset from section 4.2.1 with the following ranges:

- $m_4^2 \in [10^0, 10^{3.3}] \text{ eV}^2$
- $|U_{e4}|^2 \in [10^{-4.2}, 10^{-0.8}]$

and plot a heat map with the obtained $\Delta\chi^2$ values and the sensitivity contours for the $1\text{-}\sigma$ and 95 % C.L. What do the contour lines mean?

What happens if you introduce a sterile neutrino signature by hand? Create a dataset similar to the Asimov dataset from section 4.2.1, but with a sterile signature:

- $m_4^2 = 100 \text{ eV}^2$
- $|U_{e4}|^2 = 0.002$

and again draw the $1\text{-}\sigma$ and 95 % contour lines. What is the difference to the results obtained above from the Asimov data set without a sterile signature? Look at and compare all the different χ^2 values, what do you observe? How could one improve the results? What would be different in the real KATRIN experiment?

References

- [1] C. L. Cowan Jr. et al. “Detection of the Free Neutrino: A Confirmation”. In: *Science* 124 (1956), pp. 103–104. DOI: [10.1126/science.124.3212.103](https://doi.org/10.1126/science.124.3212.103).
- [2] Y. Fukuda et al. “Evidence for Oscillation of Atmospheric Neutrinos”. In: *Phys. Rev. Lett.* 81 (8 1998), pp. 1562–1567. DOI: [10.1103/PhysRevLett.81.1562](https://doi.org/10.1103/PhysRevLett.81.1562). URL: <https://link.aps.org/doi/10.1103/PhysRevLett.81.1562>.
- [3] Q. R. Ahmad et al. “Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory”. In: *Phys. Rev. Lett.* 89 (1 2002), p. 011301. DOI: [10.1103/PhysRevLett.89.011301](https://doi.org/10.1103/PhysRevLett.89.011301). URL: <https://link.aps.org/doi/10.1103/PhysRevLett.89.011301>.
- [4] S. Abe et al. “Precision Measurement of Neutrino Oscillation Parameters with KamLAND”. In: *Phys. Rev. Lett.* 100 (22 2008), p. 221803. DOI: [10.1103/PhysRevLett.100.221803](https://doi.org/10.1103/PhysRevLett.100.221803). URL: <https://link.aps.org/doi/10.1103/PhysRevLett.100.221803>.
- [5] B. Pontecorvo. “Mesonium and anti-mesonium”. In: *Sov. Phys. JETP* 6 (1957). [Zh. Eksp. Teor. Fiz.33,549(1957)], p. 429.
- [6] Ziro Maki, Masami Nakagawa, and Shoichi Sakata. “Remarks on the Unified Model of Elementary Particles”. In: *Progress of Theoretical Physics* 28.5 (1962), pp. 870–880. DOI: [10.1143/PTP.28.870](https://doi.org/10.1143/PTP.28.870). eprint: [/oup/backfile/content_public/journal/ptp/28/5/10.1143/ptp.28.870/2/28-5-870.pdf](https://arxiv.org/abs/1905.08702). URL: <http://dx.doi.org/10.1143/PTP.28.870>.
- [7] Steen Hannestad. “Neutrino physics from precision cosmology”. In: *Progress in Particle and Nuclear Physics* 65.2 (2010), pp. 185–208. ISSN: 0146-6410. DOI: <https://doi.org/10.1016/j.ppnp.2010.07.001>. URL: <http://www.sciencedirect.com/science/article/pii/S0146641010000499>.
- [8] Steven R. Elliott and Petr Vogel. “DOUBLE BETA DECAY”. In: *Annual Review of Nuclear and Particle Science* 52.1 (2002), pp. 115–151. DOI: [10.1146/annurev.nucl.52.050102.090641](https://doi.org/10.1146/annurev.nucl.52.050102.090641). URL: <https://doi.org/10.1146/annurev.nucl.52.050102.090641>.
- [9] KATRIN Collaboration† et al. “Direct neutrino-mass measurement based on 259 days of KATRIN data”. In: *Science* 388.6743 (2025), pp. 180–185. DOI: [10.1126/science.adq9592](https://doi.org/10.1126/science.adq9592). eprint: <https://www.science.org/doi/pdf/10.1126/science.adq9592>. URL: <https://www.science.org/doi/abs/10.1126/science.adq9592>.
- [10] M. Goldhaber, L. Grodzins, and A. W. Sunyar. “Helicity of Neutrinos”. In: *Phys. Rev.* 109 (1958), pp. 1015–1017. DOI: [10.1103/PhysRev.109.1015](https://doi.org/10.1103/PhysRev.109.1015).
- [11] A. Boyarsky et al. “Sterile neutrino Dark Matter”. In: *Progress in Particle and Nuclear Physics* 104 (2019), pp. 1–45. ISSN: 0146-6410. DOI: <https://doi.org/10.1016/j.ppnp.2018.07.004>. URL: <https://www.sciencedirect.com/science/article/pii/S0146641018300711>.
- [12] “Precision electroweak measurements on the Z resonance”. In: *Physics Reports* 427.5 (2006), pp. 257–454. ISSN: 0370-1573. DOI: <https://doi.org/10.1016/j.physrep.2005.12.006>. URL: <https://www.sciencedirect.com/science/article/pii/S0370157305005119>.
- [13] G. Mention et al. “Reactor antineutrino anomaly”. In: *Phys. Rev. D* 83 (7 Apr. 2011), p. 073006. DOI: [10.1103/PhysRevD.83.073006](https://doi.org/10.1103/PhysRevD.83.073006). URL: <https://link.aps.org/doi/10.1103/PhysRevD.83.073006>.
- [14] R.E. Shrock. “New tests for and bounds on neutrino masses and lepton mixing”. In: *Physics Letters B* 96.1 (1980), pp. 159–164. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(80\)90235-X](https://doi.org/10.1016/0370-2693(80)90235-X). URL: <https://www.sciencedirect.com/science/article/pii/037026938090235X>.
- [15] S. Mertens et al. “Sensitivity of next-generation tritium beta-decay experiments for keV-scale sterile neutrinos”. In: *Journal of Cosmology and Astroparticle Physics* 2015.02 (Feb. 2015), p. 020. DOI: [10.1088/1475-7516/2015/02/020](https://doi.org/10.1088/1475-7516/2015/02/020). URL: <https://doi.org/10.1088/1475-7516/2015/02/020>.
- [16] KATRIN Collaboration. “Sterile-neutrino search based on 259 days of KATRIN data”. In: *Nature* 648 (2025), pp. 70–75. DOI: [10.1038/s41586-025-09739-9](https://doi.org/10.1038/s41586-025-09739-9). URL: <https://www.nature.com/articles/s41586-025-09739-9>.

-
- [17] Ch Kraus et al. “Final results from phase II of the Mainz neutrino mass search in tritium β decay”. In: *The European Physical Journal C - Particles and Fields* 40.4 (2005), pp. 447–468. ISSN: 1434-6052. DOI: [10.1140/epjc/s2005-02139-7](https://doi.org/10.1140/epjc/s2005-02139-7). URL: <https://doi.org/10.1140/epjc/s2005-02139-7>.
- [18] V. N. Aseev et al. “Upper limit on the electron antineutrino mass from the Troitsk experiment”. In: *Phys. Rev. D* 84 (11 2011), p. 112003. DOI: [10.1103/PhysRevD.84.112003](https://link.aps.org/doi/10.1103/PhysRevD.84.112003). URL: <https://link.aps.org/doi/10.1103/PhysRevD.84.112003>.
- [19] KATRIN Collaboration. “The design, construction, and commissioning of the KATRIN experiment”. In: *Journal of Instrumentation* 16.08 (2021), T08015. DOI: [10.1088/1748-0221/16/08/T08015](https://doi.org/10.1088/1748-0221/16/08/T08015).
- [20] M. Kleesiek et al. *β -Decay Spectrum, Response Function and Statistical Model for Neutrino Mass Measurements with the KATRIN Experiment*. 2018. eprint: [arXiv:1806.00369](https://arxiv.org/abs/1806.00369).
- [21] E W Otten and C Weinheimer. “Neutrino mass limit from tritium β decay”. In: *Reports on Progress in Physics* 71.8 (2008), p. 086201. URL: <http://stacks.iop.org/0034-4885/71/i=8/a=086201>.
- [22] Martin Babutzka. “Design and development for the Rearsection of the KATRIN experiment”. PhD thesis. 2014.
- [23] Florian Heizmann, Hendrik Seitz-Moskaliuk, and KATRIN collaboration. “The Windowless Gaseous Tritium Source (WGTS) of the KATRIN experiment”. In: *Journal of Physics: Conference Series* 888.1 (2017), p. 012071. URL: <http://stacks.iop.org/1742-6596/888/i=1/a=012071>.
- [24] Susanne Mertens. “Study of Background Processes in the Electrostatic Spectrometers of the KATRIN Experiment”. PhD thesis. 2012.
- [25] Martin Slezák. “Monitoring of the energy scale in the KATRIN neutrino experiment”. PhD thesis. Charles University in Prague, 2015. URL: http://www.katrin.kit.edu/publikationen/phd-Martin_Slezak.pdf.
- [26] Lisa Schlüter. “Neutrino-Mass Analysis with sub-eV Sensitivity and Search for Light Sterile Neutrinos with the KATRIN Experiment”. en. PhD thesis. Technische Universität München, 2022, p. 211.
- [27] V.M. Lobashev and P.E. Spivak. “A method for measuring the electron antineutrino rest mass”. In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 240.2 (1985), pp. 305–310. ISSN: 0168-9002. DOI: [10.1016/0168-9002\(85\)90640-0](https://doi.org/10.1016/0168-9002(85)90640-0). URL: <https://www.sciencedirect.com/science/article/pii/0168900285906400>.
- [28] A. Picard et al. “A solenoid retarding spectrometer with high resolution and transmission for keV electrons”. In: *Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms* 63.3 (1992), pp. 345–358. ISSN: 0168-583X. DOI: [10.1016/0168-583X\(92\)95119-C](https://doi.org/10.1016/0168-583X(92)95119-C). URL: <https://www.sciencedirect.com/science/article/pii/0168583X9295119C>.
- [29] G Beamson, H Q Porter, and D W Turner. “The collimating and magnifying properties of a superconducting field photoelectron spectrometer”. In: *Journal of Physics E: Scientific Instruments* 13.1 (Jan. 1980), p. 64. DOI: [10.1088/0022-3735/13/1/018](https://doi.org/10.1088/0022-3735/13/1/018). URL: <https://dx.doi.org/10.1088/0022-3735/13/1/018>.
- [30] J.F. Amsbaugh et al. “Focal-plane detector system for the KATRIN experiment”. In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 778 (2015), pp. 40–60. ISSN: 0168-9002. DOI: <https://doi.org/10.1016/j.nima.2014.12.116>. URL: <http://www.sciencedirect.com/science/article/pii/S0168900215000236>.
- [31] Alessandro Schwemmer. “Direct Neutrino Mass Measurement with the KATRIN Experiment: A Frequentist and Bayesian Analysis of the First Five Measurement Campaigns”. Dissertation, embargo until Nov 18, 2026. PhD thesis. Technische Universität München, 2025. URL: <https://hdl.handle.net/21.11116/0000-0012-5975-5>.

-
- [32] Samuel S Wilks. “The large-sample distribution of the likelihood ratio for testing composite hypotheses”. In: *The Annals of Mathematical Statistics* 9.1 (1938), pp. 60–62. DOI: [10.1214/aoms/1177732360](https://doi.org/10.1214/aoms/1177732360).